Exam Kaleidoscope Modern Physics

2 december 2014, 18:30-21:30, A. Jacobshal

- Put your name and student number on each sheet.
- Answer all questions short and to the point, but complete; write legible.
- Use of a calculator is not allowed.
- $hc = 1240$ eV·nm ; $\hbar c = 200$ eV·nm .
- Final grade = total number of points/3 + 1

1. What is the kinetic energy of an electron ejected from a sodium surface whose work function is $W_0 = 2.28\,\mathrm{eV}$ when illuminated by light of wavelength (a) 410 nm (b) 550 nm?

   (2 points)

   First need the energy for each of the two photons, $E = hc/\lambda$. Since $hc = 1240\,\mathrm{eV}\cdot\mathrm{nm}$, the photon energies are (a) $3\,\mathrm{eV}$ and (b) $2.25\,\mathrm{eV}$. So in the first case the kinetic energy is $K = E - W_0 = 3 - 2.28\,\mathrm{eV} = 0.72\,\mathrm{eV}$. In the second case, the energy is too low to emit electrons. 1 point for E-lambda relation; 1/2 point for the calculations and 1/2 for the conclusion that no electron is emitted in second case. If for (b) a very small energy is calculated due to rounding errors, that is also OK.

2. The quantum states of an electron in an atom are characterised by the four quantum numbers, n, l, m_l and m_s. In a many electron atom, how many electrons can occupy the same shell, *i.e.* share the same value of n? *Hint*: first consider a combination (n, l). **(3 points)**

   For a given n, l can take the values $[0, n]$ and for a given l, m_l can take the values $[-l, l]$. For each combination n, l, m_l m_s can take the values $-1/2$ and $+1/2$. No two electrons may share the same values for all quantum numbers (Pauli exclusion principle). Each combination of quantum numbers thus contains one electron. Hence the total number of electrons is equal to the number of possible states. For a combination (n, l) the number of electrons is thus $2(2l + 1)$. Summing over $l = 0 \cdots n - 1$ gives $2n^2$ electrons per shell. Exclusion principle/1 electron per set of QN: 1 point; l = 0 ... n-1 , ml = -l ... l, ms = -1/2, 1/2: 1/2 point each; 1/2 point for final calculation

3. A radioactive element undergoes an alpha decay with a lifetime of $12\,\mu\mathrm{s}$. If alpha particles are emitted with kinetic energy $E = 5.5\,\mathrm{keV}$, find the uncertainty $\Delta E/E$ in the particle energy. Using J·s as the unit for $\hbar$ in not allowed! **(2 points)**

   Uncertainty relation: $\Delta E \Delta t \geq \hbar/2$. Multiply both sides with c to be able to use $\hbar \cdot c$ on the RHS. So we have $\Delta E \Delta t \geq \hbar/2$ or $\Delta E = \hbar/2\Delta t = \hbar \cdot c/2\Delta t \cdot c$.

4. Explain what ionic, covalent and metallic molecular bonds are. **(2 points)**

   Ionic: electrons are transferred from one atom to another, so that two ions are formed, electron spend most time near one or the other nucleus; covalent: electrons are shared between the two nuclei; metallic: electrons are shared between many atoms.

5. Excimer lasers make use of molecules consisting of an excited noble gas and a halogen, *e.g.* Ar*Cl. (a) Explain why a chlorine atom and an argon atom, both in their lowest energy state, can *not* form an ArCl molecule. (b) However, if one of the least bound electrons of Ar is promoted to the next energy state (the atom is "excited", Ar*) it *is* possible to create an Ar*Cl molecule. Why? **(3 points)**

   (a) Neon is a noble gas, with a full outer shell. The Neon electron is very strongly bound. Transfering an electron to Cl is thus energetically unfavorable. (b) if the neon atom is excited, the remaining electrons exhibit a fluorine configuration, with the last electron in an excited state. Hence a covalent bond becomes possible. 1 point for (a), 2 for (b)

6. When a proton and an antiproton (both at rest) annihilate, at least two photons are produced. Why not one? What are the wavelengths of the two photons? **(2 points)**

   The production of just a single photon is prevented by momentum conservation. All the mass of the proton and antiproton are converted to energy. The mass of an antiproton is equal to that of a proton, $m = 938\,\mathrm{MeV/c^2}$. Each photon thus has an energy of 938 MeV, which corresponds to a wavelength of $\lambda = hc/E = 200\,\mathrm{MeV}\cdot\mathrm{fm}/938\,\mathrm{MeV} \simeq 0.2\,\mathrm{fm}$.

7. Which two interactions play a significant role in the formation of an atomic nucleus? Indicate whether these interactions are attractive, repulsive or absent for pp, pn and nn nucleon pairs. What is the range of each of these forces? Use these observations to explain why large stable nuclei generally have more neutrons than protons. **(3 points)**
   The strong nuclear and EM interactions. The strong interaction is attractive for all pairs, the EM repulsive for a pp pair, and absent for pn and nn because the neutron has no electric charge. The strong force has only a limited range, whereas the EM force has a infinite range. By adding neutrons the average distance between protons is reduced, without lowering the strong binding.

8. A naturally occurring radioactive decay sequence starts with ${}^{232}_{90}\mathrm{Th}$. The first five decays of this sequence are $\alpha, \beta^-, \beta^-, \alpha, \alpha$. Determine the resulting intermediate daughter nuclei. **(2 points)**
   In α-decay A changes by -4 and Z by -2. In β^--decay A remains constant, but Z changes by +1. So the sequence becomes

$${}^{232}_{90}\mathrm{Th} \xrightarrow{\alpha} {}^{228}_{88}\mathrm{Ra} \xrightarrow{\beta^-} {}^{228}_{89}\mathrm{Ac} \xrightarrow{\beta^-} {}^{228}_{90}\mathrm{Th} \xrightarrow{\alpha} {}^{224}_{88}\mathrm{Ra} \xrightarrow{\alpha} {}^{220}_{86}\mathrm{Rn}$$

9. Consider the fusion reactions involving deuterium and tritium.

$${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H}$$

$${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + n$$

$${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + n$$

   Which of these reactions releases the largest amount of energy? For each reaction give the (approximate) energy release. *Hint*: calculate the masses on the left and righthand side, while keeping terms in m_n and m_p. **(3 points)**
   From the appendix, the approximate binding energy *per nucleon* is ${}^{2}_{1}\mathrm{H} : 1.1\,\mathrm{MeV}$, ${}^{3}_{1}\mathrm{H} : 2.9\,\mathrm{MeV}$, ${}^{3}_{2}\mathrm{He} : 2.5\,\mathrm{MeV}$, and ${}^{4}_{2}\mathrm{He} : 7.2\,\mathrm{MeV}$. Total binding on the LHS is 2*1.1+2*1.1=4.4, 4,4 and 2*1.1+3*2.9=10.9 MeV. For the RHS this becomes 3*2.9=8.7, 3*2.5=7.5 and 4*7.2=28.8 MeV. This brings the energy release to 8.7-4.4=4.3, 7.5-4.4=3.1 and 28.8-10.9=17.9 MeV.

10. Fermi problem: How many grains of rice are consumed daily in China? **(5 points)**
   Possible solution: number of people in China, 1 billion; rice consumption per person per day, e.g. volume of about 1 cup per meal, 2 meals per day per person, on average; volume of a rice grain, via size of grain (5 mm long, 2 mm diameter). Alternatively via weight. Wikipedia: 156 million metric ton per year

Fundamental Constants

Quantity	Symbol	Approximate Value	Current Best Value[†]
Speed of light in vacuum	c	3.00×10^8 m/s	2.99792458×10^8 m/s
Gravitational constant	G	6.67×10^{-11} N·m²/kg²	$6.6728(67) \times 10^{-11}$ N·m²/kg²
Avogadro's number	N_A	6.02×10^{23} mol^{-1}	$6.02214179(30) \times 10^{23}$ mol^{-1}
Gas constant	R	8.314 J/mol·K = 1.99 cal/mol·K = 0.0821 L·atm/mol·K	8.314472(15) J/mol·K
Boltzmann's constant	k	1.38×10^{-23} J/K	$1.3806504(24) \times 10^{-23}$ J/K
Charge on electron	e	1.60×10^{-19} C	$1.602176487(40) \times 10^{-19}$ C
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m²·K⁴	$5.670400(40) \times 10^{-8}$ W/m²·K⁴
Permittivity of free space	$\epsilon_0 = (1/c^2\mu_0)$	8.85×10^{-12} C²/N·m²	$8.854187817\ldots \times 10^{-12}$ C²/N·m²
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T·m/A	$1.2566370614\ldots \times 10^{-6}$ T·m/A
Planck's constant	h	6.63×10^{-34} J·s	$6.62606896(33) \times 10^{-34}$ J·s
Electron rest mass	m_e	9.11×10^{-31} kg = 0.000549 u = 0.511 MeV/c^2	$9.10938215(45) \times 10^{-31}$ kg $= 5.4857990943(23) \times 10^{-4}$ u
Proton rest mass	m_p	1.6726×10^{-27} kg = 1.00728 u = 938.27 MeV/c^2	$1.672621637(83) \times 10^{-27}$ kg = 1.00727646677(10) u
Neutron rest mass	m_n	1.6749×10^{-27} kg = 1.008665 u = 939.57 MeV/c^2	$1.674927211(84) \times 10^{-27}$ kg = 1.00866491597(43) u
Atomic mass unit (1 u)		1.6605×10^{-27} kg = 931.49 MeV/c^2	$1.660538782(83) \times 10^{-27}$ kg = 931.494028(23) MeV/c^2

[†] CODATA (3/07), Peter J. Mohr and Barry N. Taylor, National Institute of Standards and Technology. Numbers in parentheses indicate one-standard-deviation experimental uncertainties in final digits. Values without parentheses are exact (i.e., defined quantities).

Other Useful Data

Joule equivalent (1 cal)	4.186 J
Absolute zero (0 K)	−273.15°C
Acceleration due to gravity at Earth's surface (avg.)	9.80 m/s² (= g)
Speed of sound in air (20°C)	343 m/s
Density of air (dry)	1.29 kg/m³
Earth: Mass	5.98×10^{24} kg
Radius (mean)	6.38×10^3 km
Moon: Mass	7.35×10^{22} kg
Radius (mean)	1.74×10^3 km
Sun: Mass	1.99×10^{30} kg
Radius (mean)	6.96×10^5 km
Earth–Sun distance (mean)	149.6×10^6 km
Earth–Moon distance (mean)	384×10^3 km

The Greek Alphabet

Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ϵ, ε	Rho	P	ρ
Zeta	Z	ζ	Sigma	Σ	σ
Eta	H	η	Tau	T	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ, φ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Values of Some Numbers

$\pi = 3.1415927$

$\sqrt{2} = 1.4142136$

$\ln 2 = 0.6931472$

$\log_{10} e = 0.4342945$

$e = 2.7182818$

$\sqrt{3} = 1.7320508$

$\ln 10 = 2.3025851$

1 rad = 57.2957795°

Mathematical Signs and Symbols

$\propto$	is proportional to	$\leq$	is less than or equal to
$=$	is equal to	$\geq$	is greater than or equal to
$\approx$	is approximately equal to	Σ	sum of
$\neq$	is not equal to	$\bar{x}$	average value of x
$>$	is greater than	Δx	change in x
$\gg$	is much greater than	$\Delta x \to 0$	Δx approaches zero
$<$	is less than	$n!$	$n(n-1)(n-2)\ldots(1)$
$\ll$	is much less than		

Properties of Water

Density (4°C)	1.000×10^3 kg/m³
Heat of fusion (0°C)	333 kJ/kg (80 kcal/kg)
Heat of vaporization (100°C)	2260 kJ/kg (539 kcal/kg)
Specific heat (15°C)	4186 J/kg·C° (1.00 kcal/kg·C°)
Index of refraction	1.33

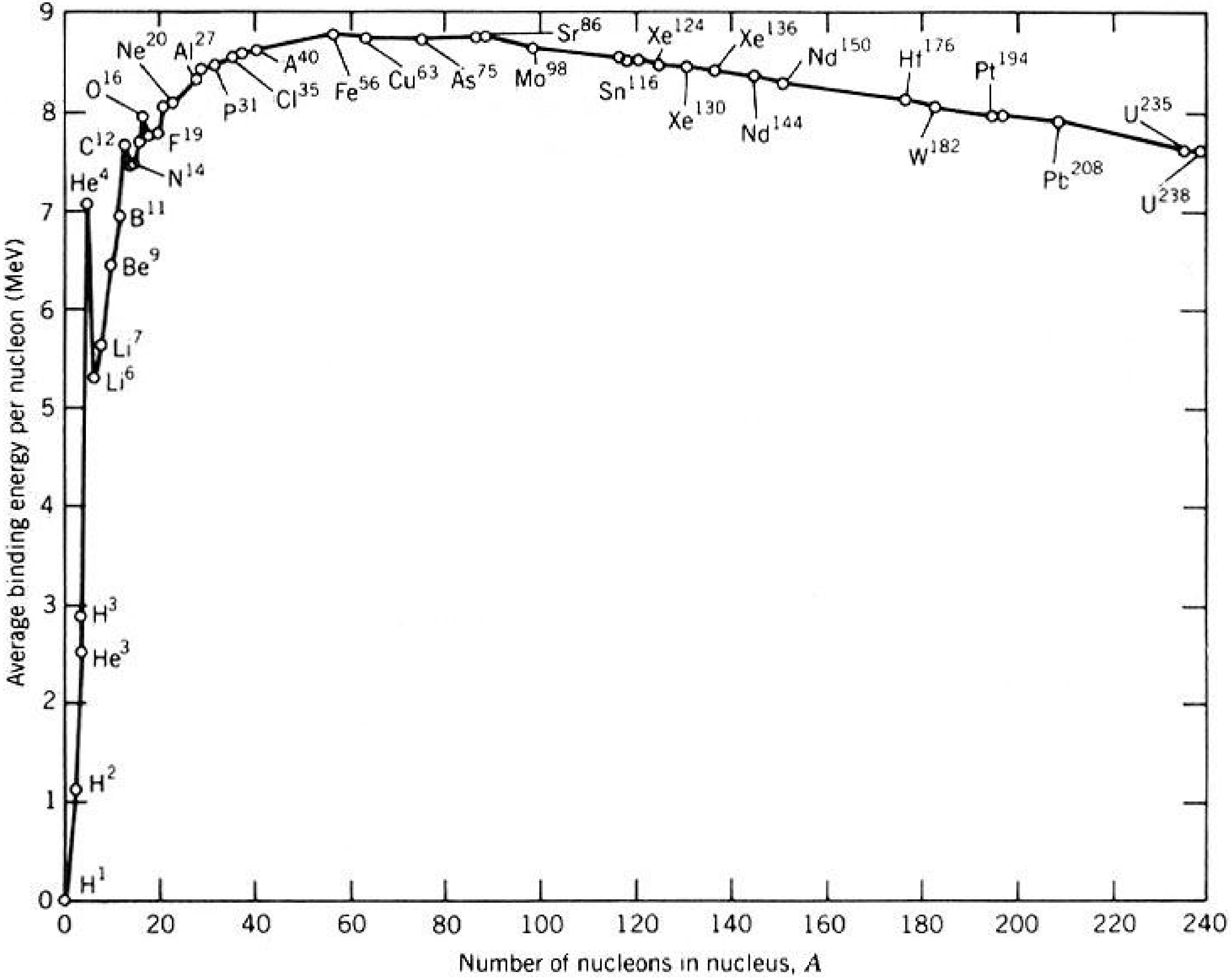
Periodic Table of the Elements[§]

Symbol — **Cl** 17 — Atomic Number
Atomic Mass§ — 35.453
$3p^5$ — Electron Configuration (outer shells only)

Group I	Group II	Transition Elements										Group III	Group IV	Group V	Group VI	Group VII	Group VIII
H 1 1.00794 $1s^1$																	**He** 2 4.002602 $1s^2$
Li 3 6.941 $2s^1$	**Be** 4 9.012182 $2s^2$											**B** 5 10.811 $2p^1$	**C** 6 12.0107 $2p^2$	**N** 7 14.0067 $2p^3$	**O** 8 15.9994 $2p^4$	**F** 9 18.9984032 $2p^5$	**Ne** 10 20.1797 $2p^6$
Na 11 22.98976928 $3s^1$	**Mg** 12 24.3050 $3s^2$											**Al** 13 26.9815386 $3p^1$	**Si** 14 28.0855 $3p^2$	**P** 15 30.973762 $3p^3$	**S** 16 32.065 $3p^4$	**Cl** 17 35.453 $3p^5$	**Ar** 18 39.948 $3p^6$
K 19 39.0983 $4s^1$	**Ca** 20 40.078 $4s^2$	**Sc** 21 44.955912 $3d^14s^2$	**Ti** 22 47.867 $3d^24s^2$	**V** 23 50.9415 $3d^34s^2$	**Cr** 24 51.9961 $3d^54s^1$	**Mn** 25 54.938045 $3d^54s^2$	**Fe** 26 55.845 $3d^64s^2$	**Co** 27 58.933195 $3d^74s^2$	**Ni** 28 58.6934 $3d^84s^2$	**Cu** 29 63.546 $3d^{10}4s^1$	**Zn** 30 65.409 $3d^{10}4s^2$	**Ga** 31 69.723 $4p^1$	**Ge** 32 72.64 $4p^2$	**As** 33 74.92160 $4p^3$	**Se** 34 78.96 $4p^4$	**Br** 35 79.904 $4p^5$	**Kr** 36 83.798 $4p^6$
Rb 37 85.4678 $5s^1$	**Sr** 38 87.62 $5s^2$	**Y** 39 88.90585 $4d^15s^2$	**Zr** 40 91.224 $4d^25s^2$	**Nb** 41 92.90638 $4d^45s^1$	**Mo** 42 95.94 $4d^55s^1$	**Tc** 43 (98) $4d^55s^2$	**Ru** 44 101.07 $4d^75s^1$	**Rh** 45 102.90550 $4d^85s^1$	**Pd** 46 106.42 $4d^{10}5s^0$	**Ag** 47 107.8682 $4d^{10}5s^1$	**Cd** 48 112.411 $4d^{10}5s^2$	**In** 49 114.818 $5p^1$	**Sn** 50 118.710 $5p^2$	**Sb** 51 121.760 $5p^3$	**Te** 52 127.60 $5p^4$	**I** 53 126.90447 $5p^5$	**Xe** 54 131.293 $5p^6$
Cs 55 132.9054519 $6s^1$	**Ba** 56 137.327 $6s^2$	57–71†	**Hf** 72 178.49 $5d^26s^2$	**Ta** 73 180.94788 $5d^36s^2$	**W** 74 183.84 $5d^46s^2$	**Re** 75 186.207 $5d^56s^2$	**Os** 76 190.23 $5d^66s^2$	**Ir** 77 192.217 $5d^76s^2$	**Pt** 78 195.084 $5d^96s^1$	**Au** 79 196.966569 $5d^{10}6s^1$	**Hg** 80 200.59 $5d^{10}6s^2$	**Tl** 81 204.3833 $6p^1$	**Pb** 82 207.2 $6p^2$	**Bi** 83 208.98040 $6p^3$	**Po** 84 (209) $6p^4$	**At** 85 (210) $6p^5$	**Rn** 86 (222) $6p^6$
Fr 87 (223) $7s^1$	**Ra** 88 (226) $7s^2$	89–103‡	**Rf** 104 (267) $6d^27s^2$	**Db** 105 (268) $6d^37s^2$	**Sg** 106 (271) $6d^47s^2$	**Bh** 107 (272) $6d^57s^2$	**Hs** 108 (277) $6d^67s^2$	**Mt** 109 (276) $6d^77s^2$	**Ds** 110 (281) $6d^97s^1$	**Rg** 111 (280) $6d^{10}7s^1$	112 (285) $6d^{10}7s^2$						

†**Lanthanide Series**	**La** 57 138.90547 $5d^16s^2$	**Ce** 58 140.116 $4f^15d^16s^2$	**Pr** 59 140.90765 $4f^35d^06s^2$	**Nd** 60 144.242 $4f^45d^06s^2$	**Pm** 61 (145) $4f^55d^06s^2$	**Sm** 62 150.36 $4f^65d^06s^2$	**Eu** 63 151.964 $4f^75d^06s^2$	**Gd** 64 157.25 $4f^75d^16s^2$	**Tb** 65 158.92535 $4f^95d^06s^2$	**Dy** 66 162.500 $4f^{10}5d^06s^2$	**Ho** 67 164.93032 $4f^{11}5d^06s^2$	**Er** 68 167.259 $4f^{12}5d^06s^2$	**Tm** 69 168.93421 $4f^{13}5d^06s^2$	**Yb** 70 173.04 $4f^{14}5d^06s^2$	**Lu** 71 174.967 $4f^{14}5d^16s^2$
‡**Actinide Series**	**Ac** 89 (227) $6d^17s^2$	**Th** 90 232.03806 $6d^27s^2$	**Pa** 91 231.03588 $5f^26d^17s^2$	**U** 92 238.0289 $5f^36d^17s^2$	**Np** 93 (237) $5f^46d^17s^2$	**Pu** 94 (244) $5f^66d^07s^2$	**Am** 95 (243) $5f^76d^07s^2$	**Cm** 96 (247) $5f^76d^17s^2$	**Bk** 97 (247) $5f^96d^07s^2$	**Cf** 98 (251) $5f^{10}6d^07s^2$	**Es** 99 (252) $5f^{11}6d^07s^2$	**Fm** 100 (257) $5f^{12}6d^07s^2$	**Md** 101 (258) $5f^{13}6d^07s^2$	**No** 102 (259) $5f^{14}6d^07s^2$	**Lr** 103 (262) $5f^{14}6d^17s^2$

[§] Atomic mass values averaged over isotopes in percentages they occur on Earth's surface. For many unstable elements, mass of the longest-lived known isotope is given in parentheses. 2006 revisions. (See also Appendix F.) Preliminary evidence (unconfirmed) has been reported for elements 113, 114, 115, 116 and 118.


Average binding energy per nucleon (MeV)
Number of nucleons in nucleus, A
0
1
2
3
4
5
6
7
8
9
0
20
40
60
80
100
120
140
160
180
200
220
240
H1
H2
He3
H3
He4
Li6
Li7
Be9
B11
C12
N14
O16
F19
Ne20
Al27
P31
Cl35
A40
Fe56
Cu63
As75
Sr86
Mo98
Sn116
Xe124
Xe130
Xe136
Nd144
Nd150
Hf176
W182
Pt194
Pt208
U235
U238